The Production of Fast Deuterons in High Energy Nuclear Reactions*

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It is proposed that the fast deuterons observed among the products of high energy nuclear reactions are to be understood in terms of a "pick-up" or sudden rearrangement process. A theoretical treatment of such a process is given which suffices to interpret presently existing data. Further experiments are proposed to test the hypothesis critically, and a possible application of this type of experiment to the determination of nuclear wave functions is pointed out.

I. INTRODUCTION

PROPORTIONAL counter experiments by York[1] and cloud-chamber measurements by Brueckner and Powell[2] have revealed a substantial number of fast deuterons among the reaction products from nuclear bombardments by 90-Mev neutrons. Analogous experiments with high energy protons seem to be giving similar results.[3] Since the energy of these deuterons is of the same order as the incident neutron energy and since they are strongly peaked forward in angular distribution, they cannot be the result of an evaporation which follows the formation of a compound nucleus. They might be understood, instead, on the basis of a sudden rearrangement, in which a proton is transferred from the target nucleus to the passing neutron. This process would be almost the inverse of deuteron stripping, which Serber[4] and Peaslee[5] have discussed.

In this paper, a calculation of the high energy "pick-up" cross section is made in Born approximation. This neglects all processes involving more than the incident nucleon and the partner with which it is to join. It is a reasonable procedure for light nuclei, which are relatively transparent to neutrons and protons with energies of the order of 100 Mev. It is hoped that any opacity may lower the over-all probability of the process without seriously changing the energy and angular dependence. Experimental data will be cited to support this point of view. The Born approximation, of course, can also fail badly in describing the elementary neutron-proton interaction, even at high energies.[6] An attempt is made to remedy this situation by separating the elementary matrix element from the rest of the calculation and making sure that this matrix element is consistent with the known facts about high energy neutron-proton scattering. The separation will become clear in Section II as the calculation is performed.

It should be pointed out that the pick-up process must not be thought of as taking place within the nucleus, following a real collision of the neutron and proton. The period of the deuteron is considerably longer than the time required for a 100-Mev neutron or proton to cross the nucleus, and the "radius" of the deuteron is as large as the mean separation of the constituents of a heavier nucleus. The "scattering" is virtual, with different energy-momentum requirements from a free neutron-proton collision, and the part of the wave function corresponding to the outgoing deuteron does not materialize as such until long after the nuclear event.

II. DERIVATION OF THE CROSS-SECTION FORMULA

It is necessary to estimate the value of a matrix element which connects an initial state consisting of an incoming free neutron, nucleus in state Ψ_i, to a final state consisting of a free deuteron, nucleus in state Ψ_f. In first approximation, we need consider only the triplet interaction between the neutron and that proton with which it is to join. Let us designate this interaction by $V(\mathbf{r}_n, \mathbf{r}_p)$ where $\mathbf{r}_n$ and $\mathbf{r}_p$ are the neutron and proton coordinates, respectively. The required matrix element is then

$$\mathcal{H}_f = (e^{i\mathbf{K}\cdot(\mathbf{r}_n+\mathbf{r}_p)/2}\varphi(\mathbf{r}_n-\mathbf{r}_p)\Psi_f,\ V(\mathbf{r}_n, \mathbf{r}_p)e^{i\mathbf{k}\cdot\mathbf{r}_n}\Psi_i) \quad (1)$$

where $\mathbf{k}$, $\mathbf{K}$ are the wave number vectors of the incident neutron and center of mass of the outgoing deuteron, respectively, and $\varphi(\mathbf{r}_n-\mathbf{r}_p)$ is the internal wave function of the deuteron. The cross section per proton for emission of a deuteron into the solid angle $d\Omega$, leaving the nucleus in the state Ψ_f, is

$$\sigma_f d\Omega = \frac{3}{4}\frac{M^2}{2\pi^2\hbar^4}\frac{K}{k}|\mathcal{H}_f|^2 d\Omega, \quad (2)$$

where M is the mass of a nucleon. The magnitude of K is determined by energy conservation:

$$\frac{\hbar^2K^2}{4M} - B_D = \frac{\hbar^2k^2}{2M} - B_{if}, \quad (3)$$

where B_{if} is the energy difference of initial and final nuclear levels and B_D is the deuteron binding energy.

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† Now at M.I.T., Cambridge, Massachusetts.
[1] H. York, Phys. Rev. **75**, 1467A (1949). A complete report of this work will be submitted for publication in the Phys. Rev.
[2] K. Brueckner and W. Powell, Phys. Rev. **75**, 1274 (1949).
[3] W. Crandall (private communication).
[4] R. Serber, Phys. Rev. **72**, 1008 (1947).
[5] D. C. Peaslee, Phys. Rev. **74**, 1001 (1948).
[6] See, for example, M. Camac and H. Bethe, Phys. Rev. **73**, 191 (1948).

If the target nucleus is very light its recoil will introduce an additional angular dependent term into (3). The factor of $\frac{3}{4}$ in (2) represents the *a priori* probability for the neutron and proton to have parallel spins.

If the relative coordinate, $\mathbf{r}=\mathbf{r}_n-\mathbf{r}_p$, is introduced in place of $\mathbf{r}_n$, the matrix element, $\mathcal{H}_f$ may be factored:

$$\mathcal{H}_f=(e^{i(\mathbf{K}-\mathbf{k})\cdot\mathbf{r}_p}\Psi_f,\,\Psi_i)(\varphi(\mathbf{r}),\,V(\mathbf{r})e^{i(\mathbf{k}-\mathbf{K}/2)\cdot\mathbf{r}}). \quad (4)$$

The final deuteron momentum, $\hbar K$, depends only weakly on the state f of the residual nucleus if the incident neutron momentum is high. We shall neglect this dependence, taking the constant value corresponding to an average B_{if} (Eq. (3)). The sum over all final nuclear states can then be performed, since

$$\sum_f |(e^{i(\mathbf{K}-\mathbf{k})\cdot\mathbf{r}_p}\Psi_f,\,\Psi_i)|^2 = \int d\mathbf{R}\left|\int e^{-(\mathbf{K}-\mathbf{k})\cdot\mathbf{r}_p}\Psi_i d\mathbf{r}_p\right|^2. \quad (5)$$

$\mathbf{R}$ represents the coordinates of all particles in the initial nucleus except the one proton which is picked up. The right-hand side of (5) can obviously be interpreted as the probability of finding the proton with the momentum, $\hbar(\mathbf{K}-\mathbf{k})$, in the initial nucleus. Let this probability be designated by $N(\mathbf{K}-\mathbf{k})$. The proton momentum in question is, of course, just that required to lead to a deuteron of momentum $\hbar\mathbf{K}$.

The second factor of (4) may be treated in two ways. To understand its meaning, let us first rewrite it as follows:

$$(\varphi(\mathbf{r}),\,V(\mathbf{r})e^{i(\mathbf{k}-\mathbf{K}/2)\cdot\mathbf{r}}) = \frac{1}{(2\pi)^3}\int d\mathbf{q}(\varphi(\mathbf{r}'),\,\exp(i\mathbf{q}\cdot\mathbf{r}')) \times(\exp(i\mathbf{q}\cdot\mathbf{r}),\,Ve^{i(\mathbf{k}-\mathbf{K}/2)\cdot\mathbf{r}}). \quad (6)$$

The second factor of the integrand on the right-hand side of (6) is essentially the probability *amplitude* for the initial neutron-proton pair to be scattered from the relative momentum, $\mathbf{k}-\mathbf{K}/2$, to the relative momentum, $\mathbf{q}$. The first factor is the probability *amplitude* for the relative momentum $\mathbf{q}$ to be found in a deuteron. Since the relative momentum is unobservable, one sums over all possible values *before* squaring to get the probability of deuteron formation. Now instead of choosing a particular interaction $V(r)$ as a starting point, one might insert for the scattering matrix element in (6) a function of $\mathbf{q}$ and $\mathbf{k}-\mathbf{K}/2$ which correctly represents the known triplet scattering cross section. This ought to eliminate much of the error of the Born approximation and also the uncertainty due to not knowing the interaction $V(r)$. Unfortunately, as mentioned above, the free neutron-proton scattering involves different pairs of momenta than occur in this problem and some kind of extrapolation has to be made. $V(r)$ can be eliminated in a different way, however, without sacrificing the desired property of the scattering matrix. The second way leaves no ambiguity as to how the extrapolation must be made.

If we remember that $V(r)$ and $\varphi(\mathbf{r})$ are connected through the Schrödinger equation for the ground state of the deuteron, then the left-hand side of (6) may be written as:

$$(V(r)\varphi(\mathbf{r}),\,e^{i(\mathbf{k}-\mathbf{K}/2)\cdot\mathbf{r}})=\left[\left(\frac{\hbar^2}{M}\Delta_r-B_D\right)\varphi(\mathbf{r}),\,e^{i(\mathbf{k}-\mathbf{K}/2)\cdot\mathbf{r}}\right].$$

The operator can be thrown back onto the exponential function, giving

$$-\left[B_D+\frac{\hbar^2}{M}(\mathbf{k}-\mathbf{K}/2)^2\right](\varphi(\mathbf{r}),\,e^{i(\mathbf{k}-\mathbf{K}/2)\cdot\mathbf{r}}) \quad (7)$$

and the neutron-proton interaction has been completely eliminated. Instead of high Fourier components of the interaction, V, one now has only to guess about high Fourier components of the deuteron wave function. There need be no concern about the nature of the force. If it is possible to choose a wave function, $\varphi(r)$, corresponding to a potential with high Fourier components which actually do agree with the observed neutron-proton scattering, then one may have confidence in the approximation. This can be done, although it actually turns out to be not very important. The two factors of (7) compensate each other to such a large extent that the function $N(\mathbf{K}-\mathbf{k})$ completely dominates the calculation.

The results of this section are that the square of the matrix element, summed over all final states of the residual nucleus, may be written as

$$\sum_f|\mathcal{H}_f|^2=N(\mathbf{K}-\mathbf{k}) \times\left[B_D+\frac{\hbar^2}{M}(\mathbf{k}-\mathbf{K}/2)^2\right]^2|(\varphi(\mathbf{r}),\,e^{i(\mathbf{k}-\mathbf{K}/2\cdot\mathbf{r})})|^2. \quad (8)$$

The unknown quantities are the initial momentum distribution of the proton to be picked up, the deuteron wave function, and the appropriate average value of B_{if}, which determines the magnitude of K.

III. DISCUSSION OF THE CROSS-SECTION FORMULA

The distribution of final energy levels is determined chiefly by the overlap of the nuclear wave functions, Ψ_i and Ψ_f. The residual nucleus is not likely to be found in its ground state, for even if the proton removed was the one most loosely bound, the wave function of the remaining nucleons will not exactly correspond to the new potential which they feel. The most probable value of B_{if}, therefore, will be greater than the energy difference between ground states of initial and final nuclei. This makes the effective threshold correspondingly higher than one would expect for an $n-d$ reaction in which a compound nucleus is formed.

An empirical average value of B_{if} can be obtained from the observed energy of the deuterons, and there may be different values for different protons within the initial nucleus. This point will be discussed further in connection with specific experiments.

The ground state of the deuteron, neglecting the small amount of d wave, may be written

$$\varphi(\mathbf{r})=A/r[e^{-\alpha r}-h(r)], \quad (9)$$

where $\alpha=[(MB_D)/\hbar^2]^{\frac{1}{2}}$, $h(r)$ is a function which is unity at the origin and vanishes outside the range of the force, and A is a normalization factor. In terms of the effective range,[7] ρ, of the triplet neutron-proton force, A may be written

$$A=\left[\frac{\alpha}{2\pi(1-\alpha\rho)}\right]^{\frac{1}{2}} \quad (10)$$

since the definition of the effective range is

$$\rho=2\int_0^\infty dr[e^{-2\alpha r}-(e^{-\alpha r}-h(r))^2]. \quad (11)$$

A simple assumption as to the form of $h(r)$ is $h(r)=e^{-\beta r}$. Then

$$\rho=4/\alpha+\beta-1/\beta. \quad (12)$$

The best value of ρ known at present is 1.6×10^{-13} cm,[7] which corresponds to $\beta=7\alpha$.

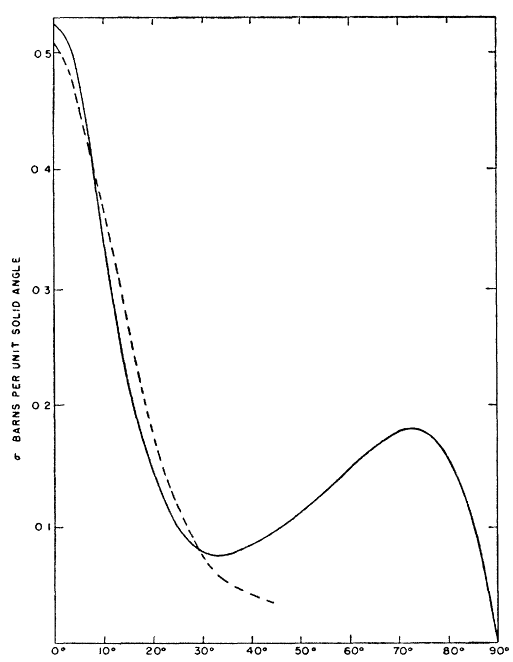

FIG. 1. Recoil deuterons from 14-Mev neutrons. Solid curve represents measurement of Coon, Taschek, and Forbes. Dotted curve follows from momentum distribution (14), normalized arbitrarily.

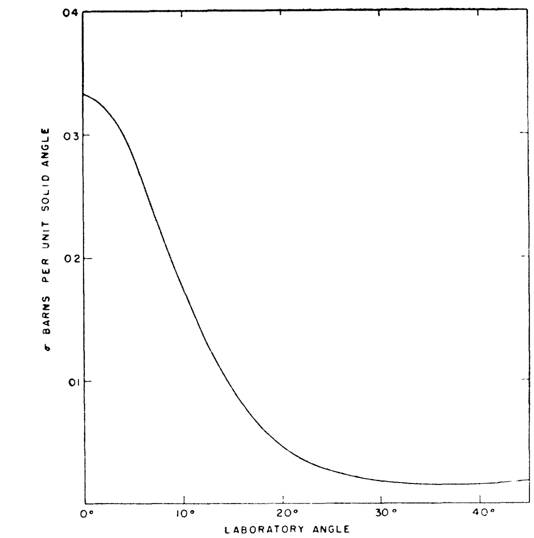

FIG. 2. Pick-up deuterons theoretically expected in the elastic scattering of 90-Mev neutrons by deuterons.

The wave function,

$$\varphi(\mathbf{r})=A(e^{-\alpha r}-e^{-\beta r})/r, \quad (13)$$

is very close to the form corresponding to a Yukawa potential.[8] It is known[8] that the Yukawa potential with a half-ordinary half-exchange character gives in Born approximation a satisfactory representation of high energy neutron-proton scattering. This is a purely fortuitous situation but is nonetheless reassuring. We believe therefore that the function (13) may be used in this particular problem with considerable confidence.

The Fourier transform of (13) is

$$4\pi A\frac{\beta^2-\alpha^2}{[\alpha^2+(\mathbf{k}-\mathbf{K}/2)^2][\beta^2+(\mathbf{k}-\mathbf{K}/2)^2]} \quad (14)$$

so that the product of the second and third factors of (8) depends on $\mathbf{K}$ only as $[\beta^2+(\mathbf{k}-\mathbf{K}/2)^2]^{-2}$. Now β^2 corresponds to about 50 Mev, while the relative energies which must be accommodated are rarely more than half this. Thus the variation of this factor over the range of interest is weak. It would be a constant if the $n-p$ force were of zero range.

The momentum distribution of the target proton, $N(k_p)$ will depend on the nucleus under bombardment. The one case where it is fairly well known is when the target nucleus is actually a deuteron. In this case, the pick-up process is simply a part of the $n-d$ elastic scattering, but it is distinguishable from the "collision part" by its forward angular distribution. In Fig. 1 is shown the angular distribution of recoil deuterons observed from 14-Mev incident neutrons.[9] The pick-up peak is evident, and the width agrees with the momentum distribution (14), even though 14 Mev is too low

[7] J. Blatt and J. D. Jackson, Phys. Rev. 76, 18 (1949).

[8] G. F. Chew, Phys. Rev. 74, 809 (1948).

[9] J. H. Coon and R. F. Taschek, Phys. Rev. 76, 710 (1949).

an energy for our calculation to apply. This is perhaps confirmation that multiple processes decrease the pick-up probability without distorting violently the angular distribution.

The corresponding experiment with 90-Mev neutrons is now being planned at the Radiation Laboratory. The theoretical prediction for the forward deuteron peak is shown in Fig. 2. This result was calculated earlier by Chew,[8] although the significance of the pick-up process was not emphasized in the earlier paper. The probability of multiple effects should be quite small in this case, so one has the opportunity of quantitatively testing the pick-up hypothesis. The expected intensity of deuterons in the forward direction is 30 mb per unit solid angle.

Experiments, however, have been and presumably will be done with targets heavier than deuterons. It is, therefore, necessary to have some idea of the momentum distribution of the protons in a complex nucleus. The simplest estimate is given by assuming the nucleus to consist of a degenerate Fermi gas of neutrons and

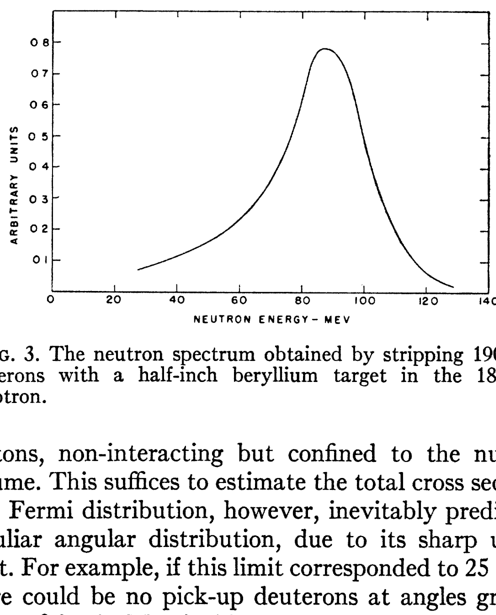

FIG. 3. The neutron spectrum obtained by stripping 190-Mev deuterons with a half-inch beryllium target in the 184-inch cyclotron.

protons, non-interacting but confined to the nuclear volume. This suffices to estimate the total cross section. The Fermi distribution, however, inevitably predicts a peculiar angular distribution, due to its sharp upper limit. For example, if this limit corresponded to 25 Mev, there could be no pick-up deuterons at angles greater than 22° for 90-Mev incident neutrons. Below this limit the angular dependence would be weak, coming entirely from the second and third factors of (8). Such a square distribution has not been observed.

In the absence of supplementary information, such as one has about the deuteron, we believe the most reasonable procedure is to read the existing proton *momentum* distribution out of the observed deuteron *angular* distribution. This same distribution should then suffice, with formula (2), to *predict* the dependence of the pick-up cross section on the *energy* of the incident neutron. The latter measurement would constitute a second critical test of the pick-up hypothesis and our treatment of it.

If these tests are passed, one would have a direct method of examining nuclear wave functions. In especially simple cases where there are only one or two neutrons or protons present so that only the first level is occupied, the energy or angular dependence of the pick-up cross section would give directly the square of the Fourier analysis of that state. These experiments should be done with monoenergetic projectiles, of course, so that the role of proton and neutron in all that has been said ought probably to be reversed.

One nucleus may be accessible even to fairly low energy measurements. This is Be^9. If there is any truth to the alpha-particle model for light nuclei, it should be possible to pick-up the odd neutron from Be^9 and still not excite the residual Be^8 by more than one or two Mev. Pick-up deuterons might therefore be produced by any accelerator which can reach a proton energy of 3 or 4 Mev. The Born approximation is not valid at low energies but the excitation function and angular distribution of these deuterons is still of interest. Be^9 should produce two distinct groups of pick-up deuterons when bombarded by protons of an energy greater than 30 or 40 Mev, since the four tightly bound neutrons could then be picked up. Again using the alpha-particle model as a guide, a B_{if} somewhat greater than 21 Mev would be expected for this second group, since this is the binding energy difference between He^3 and He^4.

IV. COMPARISON WITH EXISTING EXPERIMENTS

The most complete experimental data which is at present available is that of York.[1] He employed the neutron beam produced by deuteron stripping in the 184-inch cyclotron.[10] These neutrons are not monoenergetic, their spectrum being shown in Fig. 3, as measured in the neutron-proton scattering experiments.[11]

Most of York's results were obtained with a carbon target, for which he measured both the energy and the angular distribution of the fast deuterons. Since the dependence of the process on neutron energy has only been observed in a very crude way in this experiment,

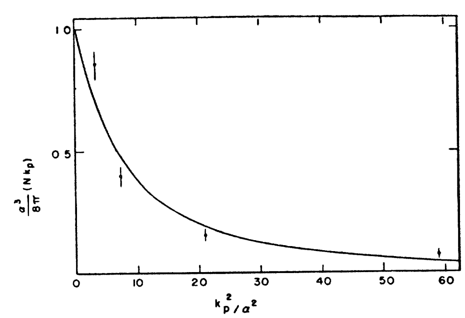

FIG. 4. Assumed distribution of proton momenta in C^{12}, normalized to unity at the origin. The four points are based on York's measurement of the deuteron intensity at 0°, 12°, 25°, 45°.

[10] Helmholz, McMillan, and Sewell, Phys. Rev. **72**, 1003 (1947).
[11] Hadley, Kelly, Leith, Segrè, Wiegand, and York, Phys. Rev. **75**, 351 (1949).

and since carbon is not very transparent, it cannot be said that the pick-up hypothesis has been verified. The most that can be done is to show that with reasonable assumptions about the carbon nucleus, York's results are understandable.

One of the most striking observations made in the $C^{12}(n, d)B^{11*}$ experiment was that the spread of possible final states in B^{11} is small. Both York and Brueckner and Powell found that the high energy end of the forward deuteron distribution gave a close reproduction of the incident neutron spectrum. It is possible to assume, therefore, that only one distinct type of proton state in the carbon is contributing, and that a single value of B_{if} is sufficient. This simplifies the problem enormously, since one can then identify the energy of a neutron by the energy of the deuteron which it produces. It is merely necessary to add 25 Mev, the difference in position of the deuteron and neutron peaks. (See Figs. 3 and 5.) According to the alpha-

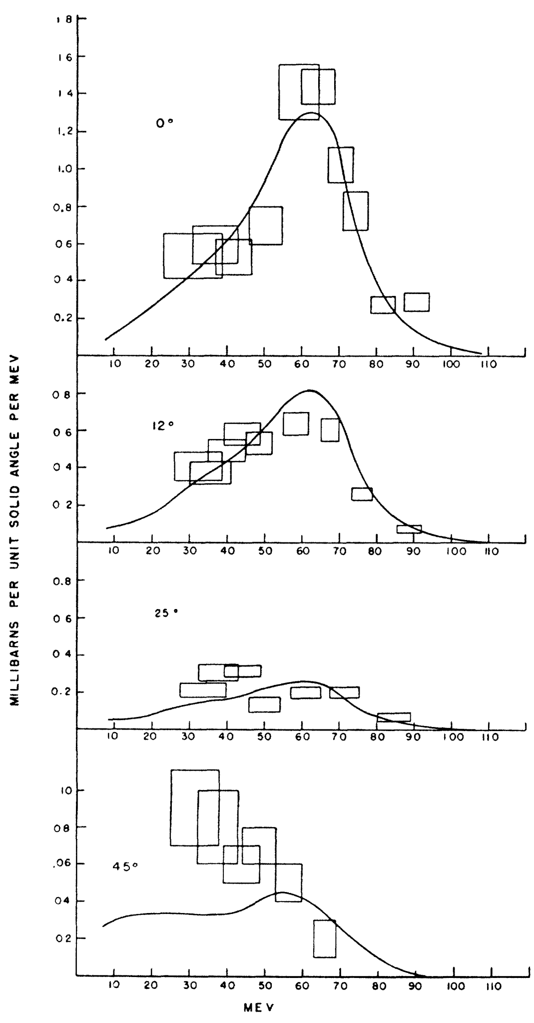

FIG. 5. Fast deuterons ejected from carbon at various angles by the stripped neutrons. Blocks represent the results of York. Solid curves are theoretical.

particle model of carbon, all six protons are equivalent, so the experimental result is perhaps not surprising.

Let us begin the analysis by determining the initial proton momentum distribution $N(k_p)$ from the angular distribution of 60–65-Mev deuterons, those at the peak. A reasonable fit over the relevant range, shown in Fig. 4, is obtained with

$$N(k_p)=\frac{8\pi\alpha_p}{(\alpha_p^2+k_p^2)^2}, \tag{15}$$

where $\mathbf{k}_p=\mathbf{K}-\mathbf{k}$, and $\alpha_p=4\alpha$. This value of α_p^2 corresponds to a proton energy of 18 Mev so that for momenta of this order of magnitude, the distribution is not surprising. At very high momenta, however, where there is as yet no evidence, this function should certainly not be believed.

Having $N(k_p)$, the problem is completely determined except for the over-all normalization. A carbon nucleus stops two-thirds of the 90-Mev neutrons entering it, so we must expect a sizeable attenuation coefficient. Choosing a value of 0.76 for the effective number of protons, and using the neutron spectrum of Fig. 3, one calculates the energy and angular distribution of deuterons shown in Fig. 5.

The agreement with experiment is satisfactory except for a group of low energy deuterons whose relative number increases with angle. These could easily be of a secondary origin, i.e., the result of interactions between three or more particles. A typical process of this type, which seems fairly likely, is to have a fast proton, produced in an exchange collision, pick up a neutron from the same nucleus. Since the incident neutron will not have lost all its energy in the initial collision, the emerging deuteron will have a smaller momentum than those considered in this paper. Such secondary deuterons should be smaller in number than the fast protons observed in the same bombardment and be less peaked in angular distribution. The data is inadequate at present to check such facts. Practically all of York's 45° deuterons could be secondary, and we may have seriously overestimated the high momentum components of the proton wave function in attempting to fit at this angle.

It should be emphasized that the agreement between theory and experiment did not follow automatically from our choice of $N(k_p)$. It is true that the heights of the deuteron peaks at various angles were forced to come out right. However, the shapes of the peaks, whose breadths increase with angle, could not be adjusted. They followed directly from the independently determined $N(k_p)$.

It is evident, nevertheless, that a really convincing experiment can only be done with a monochromatic beam, which means protons. The relatively well-defined energy of the primary pick-up deuterons should then distinguish them from secondary particles of a more complicated origin. The energy and angular dependence should also be tested in a differential rather than in an integral manner.

Two last experimental facts are worthy of mention. One is that bombardments of copper and lead targets show that the number of fast deuterons increases with atomic number less rapidly than the number of fast protons.[1] This may be an indication that the pick-up process is more confined to the surface of the nucleus than is a knock-out process. The second fact is that fast tritons have also been observed with perhaps one-tenth the probability of deuterons. If the pick-up hypothesis is correct, more complicated rearrangements are expected also to occur, but, of course, less often.

The authors wish to thank Professors Serber and Wick for helpful theoretical discussions. We are also grateful to H. York and K. Brueckner for their aid in interpreting the experiments. Work described in this report was performed under the auspices of the AEC.

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X-Ray and Gamma-Ray Reflection Properties from 500 X Units to Nine X Units of Unstressed and of Bent Quartz Plates for Use in the Two-Meter Curved-Crystal Focusing Gamma-Ray Spectrometer*

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The x-ray and gamma-ray reflection properties of the (310) planes of quartz have been investigated over the wave-length range 500 to 9 x.u. for the Laue or transmission case. The plates were inhomogeneously stressed by bending to a cylinder with a radius of 200 cm. The value of the integrated reflection coefficient was deduced from the luminosity properties of the curved-crystal spectrometer for seven different wave-lengths. The data indicate that the integrated reflection coefficient R_θ for a bent crystal varies as λ^2 over the range of wave-lengths studied. This behavior is in accord with that of a mosaic crystal. The reflection properties of the (310) planes of an unstressed crystal plate cut from the same sample were measured over the range 700 to 120 x.u. by the two-crystal spectrometer technique. These results indicate that the unstressed quartz plates behave more nearly as perfect crystals. Data are given on the integrated reflection coefficient, the peak value of Laue reflection coefficient, and the width at half-maximum of the diffraction curve for the unstressed case.

INTRODUCTION

DURING the development of the curved-crystal gamma-ray spectrometer,[1] it became apparent that a careful determination of the reflection properties of the (310) planes of the elastically curved-quartz plates used in the spectrometer would have to be made. At the same time, a thorough analysis of the intensity problem of the spectrometer was carried out. This analysis showed that the determination of the integrated Laue reflection coefficient of the curved crystal was possible from the experimental reflection properties.

While this determination is of particular interest for the design and operation of the curved-crystal spectrometer, it has additional importance because, in the past, other observers[2,3] have noted that the x-ray reflections from inhomogeneously stressed quartz plates show rather marked anomalies not present in unstressed or homogeneously stressed plates. Our experimental results confirm these qualitative observations but, in addition, we offer some quantitative data which may be useful for interpreting the observations.

THEORY

The theory of x-ray diffraction has been completely worked out for a great many conditions. From the character of the diffraction it is possible to deduce some information concerning the perfection of the lattice structure and the nature of its imperfections. A perfect lattice is one in which there exist no disorders of any kind in the atomic arrangement throughout the complete crystal. A mosaic structure, on the other hand, is one in which disorders do exist. It is convenient to describe a mosaic crystal as consisting of small domains each with perfect internal lattice structure which are more or less disarranged in the macroscopic crystal. The essential effect of the domain structure is to cause the scattering from separate domains to be incoherent. For the perfect crystals, two cases are of interest. A "thick" crystal is one for which the primary extinction distance is much smaller than the thickness of the crystal; a "thin" crystal is one for which the extinction distance is much greater than the thickness of the crystal. In a mosaic crystal, the situation is somewhat

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† Now with California Research Corporation, La Habra, California.
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[2] Y. Sakisaka and I. Sumato, Proc. Phys. Math. Soc. Japan III **13**, 211 (1931).
[3] C. S. Barrett and C. E. Howe, Phys. Rev. **39**, 889 (1932).